647510 **MSC1MatC103x** **Seat No :________**

M.Sc.(Maths.) Semester - 1 (*CBCS*) Examination

Nov/Dec - 2022 [NEW COURSE]

Topology 1(Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer any Two out of Three **[10]**

(1) Show that if τ_1 and τ_2 are two topologies on a nonempty set X then $\tau_1 \cap \tau_2$ is a topology on X, but $\tau_1 \cup \tau_2$ need not be a topology on X.

(2) Let $\mathfrak{B}_1$ and $\mathfrak{B}_2$ be basis for the topologies τ_1 and τ_2 on X respectively. Prove that τ_2 is finer than τ_1 if and only if for each $B \in \mathfrak{B}_1$ and for each $x \in B$ there exists $C \in \mathfrak{B}_2$ such that $x \in C \subset B$.

(3) Let S be a subbasis for a topology on X and B be the collection of all finite intersections of members of S. Prove that B is a basis for the topology τ generated by the subbasis S on X.

Q.1(B) Answer any One out of Two **[04]**

(1) Let X be a topological space. Suppose $\mathcal{C}$ is a collection of open sets of X such that for each open set U of X and each x in U, there exist an element $C \in \mathcal{C}$ such that $x \in C \subset U$. Then prove that $\mathcal{C}$ is a basis for the topology on X.

(2) Prove that Lower limit topology $\mathbb{R}_l$ is strictly finer than standard topology on $\mathbb{R}$.

Q.2(A) Answer any Two out of Three **[10]**

(1) Let X, Y_1, Y_2 be topological spaces and $Y_1 \times Y_2$ be product space then prove that a function $f: X \to Y_1 \times Y_2$ defined by $f(x) = \left(f_1(x), f_2(x)\right)$ for each $x \in X$ is a continuous function if and only if $f_1: X \to Y_1$ and $f_2: X \to Y_2$ are continuous functions.

(2) Prove that: Any two open intervals of $\mathbb{R}$ are homeomorphic.

(3) Let X, Y, Z be topological spaces. If both of the functions $f: X \to Y$ and $g: Y \to Z$ are continuous then prove that function $g \circ f: X \to Z$ is continuous.

Q.2(B) Answer any One out of Two **[04]**

(1) Let $f: X \to Y$ be a continuous function and let $A \subset X$. Prove that if the subspace topology of X is defined on A then the restriction function $f|_A$ of f to A is also continuous.

(2) If A is a subspace of X and B is a subspace of Y then prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace topology on $X \times Y$.

Q.3(A) Answer any Two out of Three **[10]**

(1) Let Y be a subspace of X then prove that a set A is closed in Y if and only if $A = B \cap Y$ for some closed set B in X.

(2) Let A be a subset of the topological space X and A' be the set of all limit points of A in X, then prove that $\bar{A} = A \cup A'$.

(3) State and prove: Pasting Lemma

Q.3(B) Answer any One out of Two **[04]**

(1) For a subset A and B of a topological space X, prove that $\overline{(\mathrm{X}-\mathrm{A})} = \mathrm{X} - \mathrm{Int}(\mathrm{A})$ and $\overline{(\mathrm{A} \cup \mathrm{B})} = \overline{\mathrm{A}} \cup \overline{\mathrm{B}}$.

(2) Let $f:(X,T_1) \to (Y,T_2)$ be a function. Show that the function f is continuous if and only if $f(\bar{A}) \subset \overline{f(A)}$ for each $A \subset X$.

Q.4(A) Answer any Two out of Three **[10]**

(1) Prove that the uniform topology on $\mathbb{R}^J$ is finer than the product topology.

(2) Show that $\mathbb{R} \times \mathbb{R}$ in the dictionary order topology is metrizable.

(3) State and prove: Sequence lemma.

Q.4(B) Answer any One out of Two **[04]**

(1) Show that $\mathbb{R}^w$ in the box topology is not metrizable.

(2) State and Prove: Uniform Limit Theorem.

Q.5(A) Answer any Two out of Three **[10]**

(1) Prove that every discrete space X is locally connected.

(2) For an integer $n>1$, prove that the Punctured Euclidean space $\mathbb{R}^n - \{0\}$ is path connected.

(3) State and Prove: Intermediate value theorem.

Q.5(B) Answer any One out of Two **[04]**

(1) Prove that the continuous image of a connected space is connected.

(2) Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X.

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